A homoclinic scenario of chaos in coherent atom-field dynamics

D.V.Makarov, L.E.Kon'kov, I.L.Kirilyuk, and S.V.Prants

Lab. of Nonlinear Dynamical Systems, Pacific Oceanological Inst. Rus. Ac. Sci.,
43 Baltiiskaya Street, 690041, Vladivostok, Russia
Tel+7-4232-313081, Fax+7-4232-312573
E-mail:prants@mail.ru

The simplest quantum model of the coherent atom-field dynamics comprises a collection of two-level atoms interacting with a single-mode field in a lossless cavity. In spite of its simplicity, the model is intrinsically nonlinear and describes a variety of nonlinear effects including semiclassical Hamiltonian chaos. The aim of the present communication is to find a mechanism of this chaos for two basic quantum-optical Hamiltonians which can be written in the united form as

$$H = \frac{1}{2}\hbar\omega_a \sum_{j=1}^{N} \sigma_z^j + \hbar\omega_f(a^\dagger a + \frac{1}{2}) + \hbar\Omega_0 \sum_{j=1}^{N} [(a\sigma_+^j + a^\dagger\sigma_-^j) + r(a\sigma_-^j + a^\dagger\sigma_+^j)], \quad (1)$$

where σ are the Pauli atomic operators, a and $a^\dagger$ the boson field operators. With $r = 0$ it models the integrable RWA interaction, and with $r = 1$ it takes into account the counter-rotating terms. In the Heisenberg picture, a hicrarchy of the equations of motion for the atomic and field expectation values can be closed under the condition of a semiclassical factorization. The respective set of nonlinear equations is known to be integrable with $r = 0$. The non-RWA equations with $r = 1$ demonstrate chaos which begins to show up itself when the coupling coefficient $\Omega_N \equiv \Omega_0\sqrt{N}/\omega_a$ reaches the value of the order of 0.5 [1]. We give a careful analysis of the atom-field nonlinear dynamical system and find a homoclinic structure that arises in the vicinity of a separatrix of the RWA version of (1). This separatrix forms a loop in the plane $(z, dz/d\tau)$ (where z is the density of the atomic inversion) whose stable and unstable tendrils coincide in the RWA. The perturbation caused by the counter-rotating terms in (1) gives rise to a spliting of the loop. The splitting changes its sign with the frequency $\omega_a + \omega_f$ implying transversal intersections of the stable and unstable tendrils of the unperturbed separatrix. This complicated motion is known to produce chaos of the horseshoe kind. Poincaré sections with $\Omega_N < 0.25$ demonstrate invariant orbits corresponding to periodic solutions of the integrable RWA limit. These orbits break down with increasing Ω_N (Fig. 1a). At $\Omega_N = 0.5$ chaos invades almost the whole energetically available phase plane (Fig. 1b). The regime of global chaos can be reached with $N \simeq 10^{12}$ Rydberg atoms in a high-quality microwave cavity.

It follows from our analysis that a homoclinic structure may emerge under almost all nontrivial perturbations of the separatrix loop. We have shown recently [2] that chaos may arise even in the RWA if there are physical reasons that modulate the c-number coefficients of (1) with $r = 0$. Modulation of the coupling Ω_0 arises naturally when atoms move through a spatially varied cavity mode. The RWA structure of the governing Hamiltonian allows to go beyond

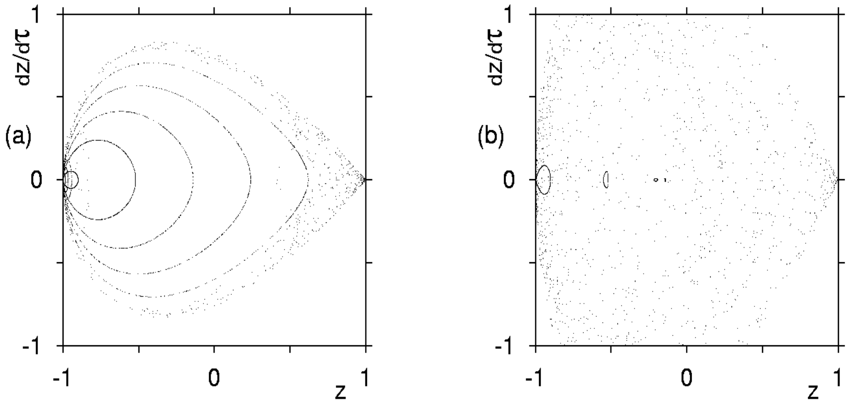

Figure 1: Poincaré sections with (a) $\Omega_N = 0.27$ and (b) $\Omega_N = 0.5$.

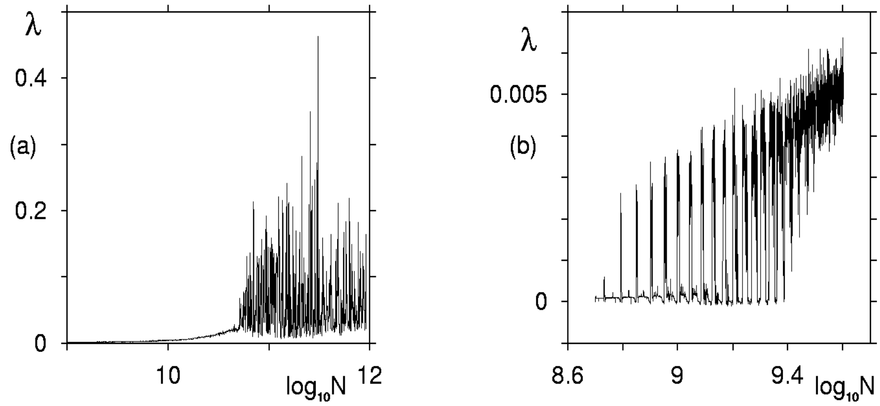

Figure 2: Dependence of the maximal Lyapunov exponent on the number of atoms on a logarithmic scale for (a) the superfluorescent initial atomic state and (b) the superradiant one.

the semiclassical approximations and describe in the framework of the Heisenberg approach the effect of vacuum Rabi oscillations caused by quantum interatomic correlations. In Fig. 2a the maximal Lyapunov exponent λ is plotted as a function of the number of atoms N with fully inverted atoms and initial vacuum field. Fig. 2b shows the same but with atoms prepared initially in a superradiant state. A curious strusture of this dependence reflects an intermittent route to global chaos. We show analytically that chaos in the vacuum Rabi oscillations is also homoclinic, but in difference from the case with atoms at rest chaos with moving atoms is structural. We find useful to study this type of chaos with the help of the wavelet analysis. The results of the wavelet transfomation of the chaotic vacuum Rabi oscillations demonstrate clearly the effect of intermittency.

Acknowledgments. This work was supported by Grant No. 99-02-17269 from the Russian Foundation for Basic Research.

[1] L. E. Kon'kov and S. V. Prants, *J. Math. Phys.* **37** 1204 (1996).
[2] S. V. Prants, L. E. Kon'kov, and I. L. Kirilyuk, *Phys. Rev. E.* **60**, 335 (1999).